Optimum Cylindrical Pin Fin

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INTRODUCTION

Cylindrical pin fins, Figure 1, are a common geometry for extended surface applications. The largest collection of information on the topic of pin fins is presented by Kays and London (1964). Recently, pin fin studies have been performed by Sparrow and his coworkers (1978, 1980) for various kinds of tube arrays and flow conditions with tip clearance. In a previous study on optimum cylindrical pin fins, Sonn and Bar-Cohen (1981) derive an expression for the diameter of the least-material cylindrical pin fin and explore the consequences of their optimization study. In their study, the fin volume or mass V_p, the temperature difference between the fin surface and the ambient at the base Θ_b, the fin thermal conductivity k, and the convective heat transfer coefficient h are fixed, and the heat transfer through the fin base is dependent only on the fin diameter D. Under more realistic heat transfer conditions, the convective heat transfer coefficient h is dependent on geometrical curvature and is inversely proportional to the diameter D^n, i.e., $h \propto 1/D^n$ (e.g., Eckert and Drake, 1972; Zukauskas, 1972). The effect of this assumption on the optimization of a single cylindrical pin fin, an in-line, and a staggered pin fin array (Figure 2) will be analyzed in this paper.

ANALYSIS

Following Kern and Kraus (1972), the fin overall heat transfer rate, or equivalently heat flow through the fin base, q_b, can be expressed as:

$$q_b = k\frac{\pi}{4}D^2\left.\frac{d\Theta}{dx}\right|_{x=b} = \frac{\pi}{4}D^2 km\Theta_b \tanh mb, \tag{1}$$

where the geometric variables are identified in Figure 1; Θ equals the temperature difference between the fin surface and the ambient, k the fin thermal conductivity, h the convective heat transfer coefficient, and $m = (4h/kD)^{1/2}$.

To determine the optimum fin dimensions, it is convenient to define a fin volume parameter, V_p, equal to D^2b, and express Eq. 1 in terms of D and V_p as

$$q_b = \frac{\pi}{2}\Theta_b(khD^3)^{1/2}\tanh\{2V_p(h/kD^5)^{1/2}\}. \tag{2}$$

From the analysis of experimental work, e.g., Zukauskas (1972); Eckert and Drake (1972) and the assumption of constant thermal fluid properties, the Nusselt number for heat transfer to tubes in cross flow can be presented in the form

$$Nu \propto Re^{1-n}, \tag{2a}$$

where the values of n are listed in Table 1 for various kinds of tube arrays and flow conditions.

From Eq. 2a, one can obtain the convection heat transfer coefficient as

$$h \propto \frac{1}{D^n}. \tag{2b}$$

For a fixed fin volume or mass, V_p is constant and, in the absence of variations in Θ_b and k, and in consideration of Eq. 2b, the heat flow through the fin base is dependent only on the diameter, D.

From Eq. 2b, Eq. 2 can be rewritten as

$$q_b = \frac{\pi}{2}\Theta_b(khD^n)^{1/2}D^{3/2-n/2}\tanh\{2V_p(D^n h/k)^{1/2}D^{-5/2-n/2}\}. \tag{2c}$$

Differentiating Eq. 2c with respect to D and evaluating at the point where the derivative vanishes, i.e.,

$$\frac{dq_b}{dD} = \frac{\pi}{2}\Theta_b(khD^n)^{1/2}\left[\left(\frac{3}{2}-\frac{n}{2}\right)D^{1/2-n/2}\tanh\{2V_p(h/kD^5)^{1/2}\} - \left(\frac{5}{2}+\frac{n}{2}\right)D^{-2-n}\{2V_p(D^n h/k)^{1/2}\}\operatorname{sech}^2\{2V_p(h/kD^5)^{1/2}\}\right] = 0 \tag{3}$$

and inserting $\beta_s = 2V_p(h/kD^5)^{1/2}$, yields

TABLE 1. VALUES OF n FOR VARIOUS FLOW CONDITIONS AND PIN FIN ARRAYS

Pin Fin \ *Re* No.	1×10^2 -1×10^3	1×10^3 -2×10^5	2×10^5 -1×10^6
Single	0.5	0.4	0.3
In-Line Array for $a/b \geq 0.7$*	0.5	0.37	0.2
Staggered Array	0.5	0.4	0.2

* $a/b < 0.7$ is considered ineffective as a heat exchanger, where a and b are defined in Figure 2.

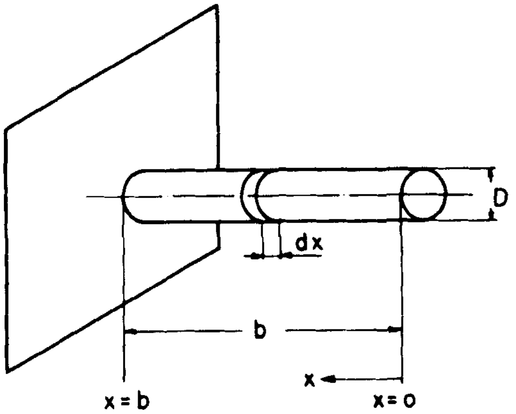

Figure 1. Cylindrical pin fin.

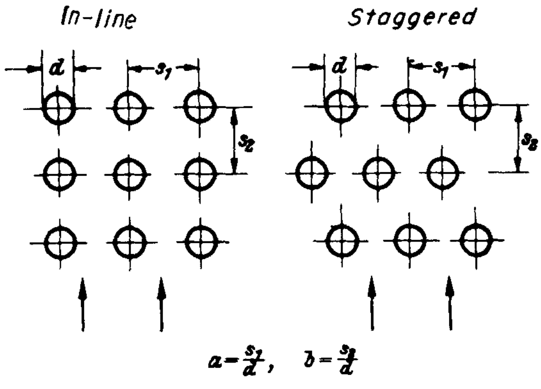

Figure 2. Bundles with in-line and staggered tubes.

TABLE 2. OPTIMUM DIAMETER, OVERALL HEAT TRANSFER RATE AND FIN EFFICIENCY OF OPTIMUM PIN FIN FOR VARIOUS n

n	0.5	0.4	0.37	0.3	0.2	0.0
β_s	1.170911	1.121655	1.106827	1.072092	1.022010	0.919296
$D_{opt}/(hV_p^2/k)^{0.2}$	1.2388	1.2603	1.2670	1.2833	1.3081	1.366
$D_{opt}/(hb^2/k)$	2.9175	3.1794	3.2651	3.4801	3.8296	4.73
$q_{opt}/(\Theta_b h^2 b^3/k)$	6.4545	7.1965	7.4414	8.0589	9.0724	11.736
η_{opt}	0.7042	0.7205	0.7254	0.7371	0.7541	0.7893

$$\tanh\beta_s = \frac{5+n}{3-n}\,\beta_s\,\mathrm{sech}^2\,\beta_s, \tag{4}$$

or following the appropriate trigonometric substitution

$$\sinh 2\beta_s = \frac{5+n}{3-n}\,(2\beta_s). \tag{5}$$

Equation 5 is a transcendental equation in β_s and can be solved with the Newton-Raphson method. The solutions of β_s for several values of n are listed in Table 2. From these β_s values one can find the optimum fin diameter D_{opt}, the overall heat transfer rate of the optimum pin fin q_{opt}, and the fin efficiency η as shown in Table 2.

The optimum mb product—a closer examination of the three fin parameters: m, V_p, and β_s—reveals that β_s is equal to mb a common parameter in fin analysis, Table 2.

The fin efficiency, η, defined as the ratio of the heat actually transferred by the fin to the heat transferred by an ideal isothermal (at the base temperature) fin, can be expressed as,

$$\eta = \frac{\tanh\,(mb)}{mb}\,.$$

The optimum fin efficiency η for various kinds of flow conditions and tube arrays are shown in Table 2.

Discussion

Unlike the work of Sonn & Bar-Cohen (1981), this study shows the values of diameter, overall heat transfer rate, mb product and fin efficiency of the optimum pin fin for various flow conditions and pin fin arrays.

In view of the similarities between this study and the work of Sonn & Bar-Cohen, it is interesting to compare the optimum performance of these two papers. The study of Sonn & Bar-Cohen (1981) is based on a single pin fin with h being constant, i.e., $n = 0$, as shown in Table 2. Since V_p is constant, in comparing the results of this study on the optimum cylindrical pin fin with Sonn and Bar-Cohen study, we find that the deviation of the optimum pin fin diameter D_{opt} is about 8.4% and the deviation of the optimum fin efficiency η_{opt} is about 9.5% for the oversimplified assumption of constant convection heat transfer coefficient.

NOTATION

b = length of fin
D = diameter of fin
h = coefficient of convective heat transfer
k = thermal conductivity of fin
m = $(4h/kD)^{1/2}$
n = constant shown in Table 1
Nu = Nusselt number
q = overall heat transfer rate
Re = Reynolds number
V_p = D^2b, volume parameter

Greek Letters

β_s = $2V_p(h/kD^5)^{1/2}$
η = fin efficiency
Θ = temperature difference between fin surface and ambient

Subscripts

b = fin base
opt = optimum
p = parameter

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